Portfolio Management Using Prediction Rules and Communication in Social Networks

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Abstract. In this study, we conduct a comprehensive analysis of Google stock (GOOGL) daily returns using the Yahoo Finance dataset. The dataset is preprocessed and partitioned into past and future segments in a 3:1 ratio, with various statistical methods applied, such as daily return calculations, autocorrelation function (ACF), partial autocorrelation function (PACF), and Dickey-Fuller stationarity testing. By constructing two Bayes detectors based on Logistic and Gaussian distributions and incorporating association rules, we develop investment strategies aimed at limiting trading frequency and reducing transaction costs. We assess the effectiveness of these strategies through simulation, taking into account factors such as risk-free interest, greed, efficient frontier, and adaptive greed. Overall, this study offers valuable insights into the daily returns of GOOGL and the potential for implementing profitable investment strategies.

Keywords: Google, Dickey-Fuller Test, Bayes Detector, Association Rules, Portfolio Management, Efficient Frontier, Adaptive Greed

1 Data Preprocessing

For the data collecting part, we choose Google [1] stock from Yahoo Finance with 6000 days in Yahoo Finance as the data source. Then we define today $t = 0$ as 2018-08-31 to split its closing-price time series $S(t)$ into two parts, "past" and "future", where the length ratio of the "past" $\{S(t)|t \leq 0\}$ to the "future" $\{S(t)|t \geq 0\}$ is $3:1$.

1.1 Daily Return

The daily return $X(t)$ of this stock is defined as follows:

$$X(t) = \ln\left[\frac{S(t)}{S(t-1)}\right] \tag{1}$$

We illustrate $S(t)$ and its daily return rate in Fig. 1 and Fig. 2 respectively. The red dashed line represents the boundary between "past" and "future."

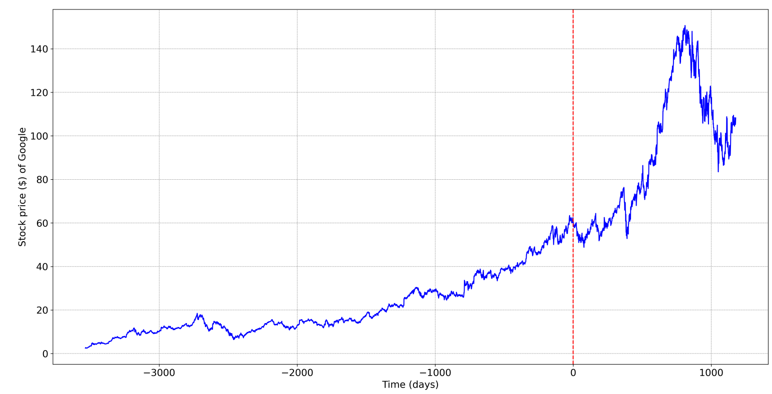

Fig. 1: Stock price $S(t)$ versus time t

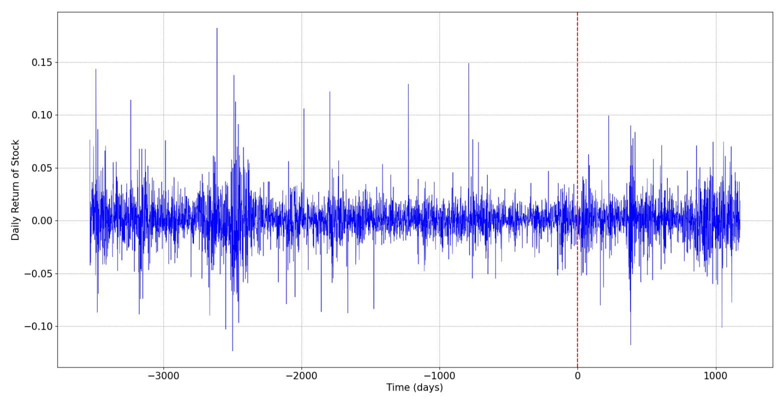

Fig. 2: Daily return $X(t)$ versus time t

1.2 Autocorrelation Function

We plot the autocorrelation function and the partial autocorrelation function of $X(t)$ in Fig. 3 and Fig. 4 respectively.

1.3 Dickey-Fuller Test

Moreover, we apply the Augmented Dickey-Fuller (ADF) test to the daily return date $X(t)$ to determine whether our time series is stationary or not. And we get,

ADF test statistic: -29.5458
P-value: 0.0000

Since P-value < 0.05, the null hypothesis is rejected. Thus, our time series is stationary.

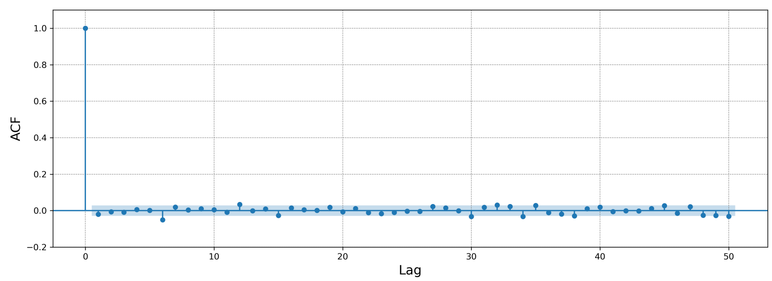

Fig. 3: Autocorrelation function of $X(t)$

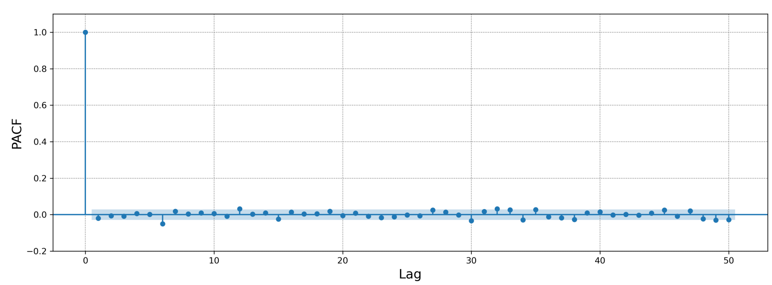

Fig. 4: Partial Autocorrelation function of $X(t)$

1.4 Digitization

We also digitize $X(t)$ as $Y(t)$ with three alphabets, viz. D for "down", U for "up", and H for "hold":

$$Y(t) = \begin{cases} D & |X(t)| < \epsilon \\ U & |X(t)| > \epsilon \\ H & otherwise \end{cases} \tag{2}$$

To accurately digitize $X(t)$ into $Y(t)$, it is crucial to select an appropriate positive constant value ϵ. Determining a suitable value for ϵ holds great significance, as it dictates the resolution of the digitization process. In the event that ϵ is excessively small, numerous minor fluctuations in $X(t)$ may be classified under the H category, thereby resulting in information loss. Conversely, if ϵ is overly large, it may cause the omission of several substantial alterations in $X(t)$, once again leading to the loss of vital information.

Especially, We use the experience equation $\epsilon = 0.28\sigma_x$ to compute ϵ where σ_x is the standard deviation of $X(t)$.The digitization result of "past" is shown as,

Epsilon: 0.0054
The proportion of U in past part: 35.34%
The proportion of D in past part: 29.94%
The proportion of H in past part: 34.72%

2 Cumulative Distribution Function

Considering the daily return $X(t)$, our objective is to predict its value $X(t+1)$ for the subsequent day. Nonetheless, a quantitative prediction proves to be overly ambitious; instead, it is more pragmatic to qualitatively predict $Y(t+1)$. We shall primarily concentrate on $Y(t+1) = \mathrm{U}$ and $Y(t+1) = \mathrm{D}$, which correspond to robust bullish and bearish markets, respectively.

The conditional Cumulative Distribution Function (CDF) of X is illustrated in Fig. 5, given that Y will transition to U one day later. This is denoted as $F_{\mathrm{U}}(x) \equiv \mathrm{CDF}[X(t) = x \mid Y(t+1) = \mathrm{U}]$.

In a similar fashion, we present the plot for $F_{\mathrm{D}}(x) \equiv \mathrm{CDF}[X(t) = x \mid Y(t+1) = \mathrm{D}]$ in Fig. 6.

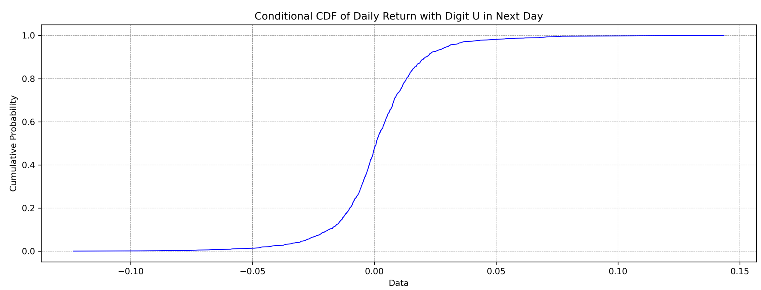

Fig. 5: Conditional CDFs of $X(t)$ given $Y(t+1) = \mathrm{U}$ in Next Day

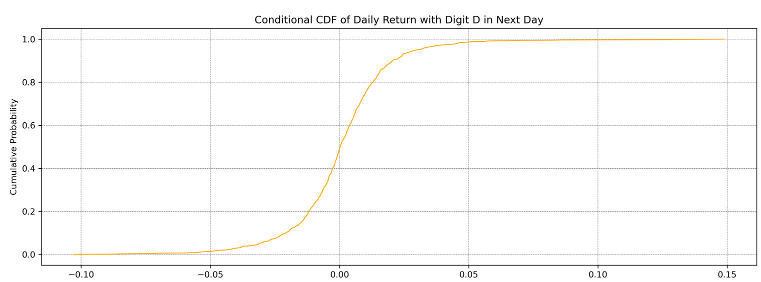

Fig. 6: Conditional CDFs of $X(t)$ given $Y(t+1) = \mathrm{D}$ in Next Day

3 Probability Density Function

Subsequently, our aim is to derive the corresponding Probability Density Functions (PDFs) denoted as $f_{\mathrm{U}}(x) \equiv \mathrm{PDF}[X(t) = x \mid Y(t+1) = \mathrm{U}]$ and $f_{\mathrm{D}}(x) \equiv \mathrm{PDF}[X(t) = x \mid Y(t+1) = \mathrm{D}]$.

3.1 Logistic distribution

On one hand, fit $F_{\mathrm{U}}(x)$ and $F_{\mathrm{D}}(x)$ with a logistic function

$$L(x) = \frac{1}{1 + \exp\left[-b\left(x - x^*\right)\right]} \tag{3}$$

where b and x^* are the fitting parameters. By fitting, we can obtain the value of b and x^* for $F_{\mathrm{U}}(x)$ and $F_{\mathrm{D}}(x)$ as follows:

- The parameters of the logistic function with digit U: b=118.2856, x*=0.0012
- The parameters of the logistic function with digit D: b=113.4866, x*=0.0005

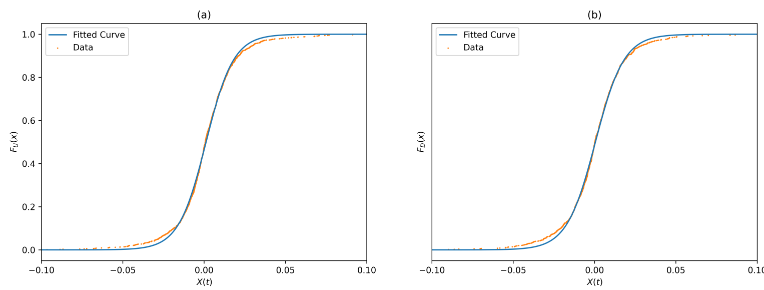

Fig. 7: Fitted with logistic functions

Subsequently, the fitted logistic functions are illustrated in Fig. 7. The derivatives of the results, namely $L'_{\mathrm{U}}(x)$ and $L'_{\mathrm{D}}(x)$, provide estimations of the desired PDFs. The derivative of the logistic function can be formulated as follows:

$$L'(x) = bL(x)(1 - L(x)) \tag{4}$$

Following this, the estimated PDFs of $X(t)$, given $Y(t+1) = \mathrm{U}$ and $Y(t+1) = \mathrm{D}$, are depicted in Fig. 8.

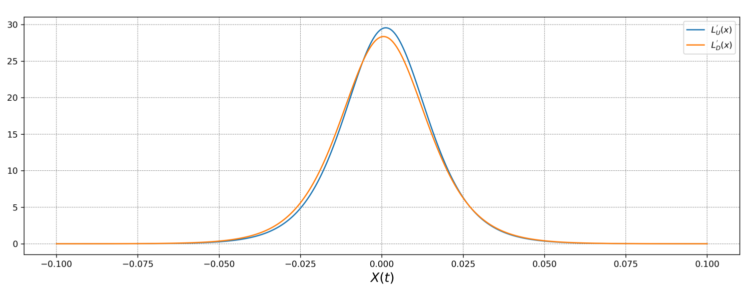

Fig. 8: $L_U^{'}(x)$ and $L_D^{'}(x)$

3.2 Gaussian distribution

On the other hand, I measure the means μ and the variances σ^2 of the data $\{X(t) = x \mid Y(t+1) = \mathrm{U}\}$ and $\{X(t) = x \mid Y(t+1) = \mathrm{D}\}$ in order to directly estimate $f_{\mathrm{U}}(x)$ and $f_{\mathrm{D}}(x)$ with a Gaussian distribution

$$g(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right].$$

By computation, we can get the value of μ and σ for $g_{\mathrm{U}}(x)$ and $g_{\mathrm{D}}(x)$ as follows:

- The parameters of Gaussian function with digit U: mu=0.0014, var=0.0004
- The parameters of Gaussian function with digit D: mu=0.0005, var=0.0004

The outcomes $g_{\mathrm{U}}(x)$ and $g_{\mathrm{D}}(x)$ are illustrated in Fig. 9.

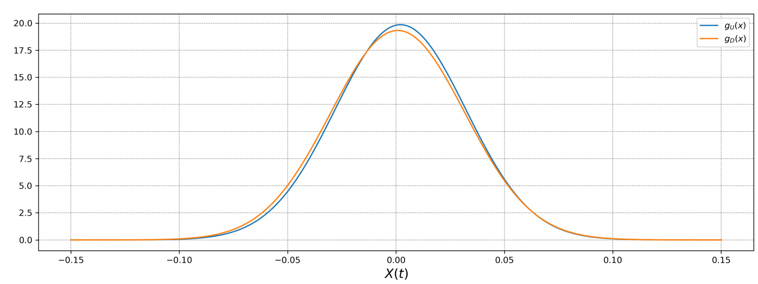

Fig. 9: $g_U(x)$ and $g_D(x)$

4 Bayes Detector

We proceed to construct a Bayes detector utilizing the PDFs to predict $Y(t+1)$ upon observing $X(t)$. Formally, we select the hypothesis " $H_{\mathrm{U}} : Y(t+1) = \mathrm{U}$ " as the null hypothesis and " $H_{\mathrm{D}} : Y(t+1) = \mathrm{D}$ " as the alternative hypothesis.

4.1 Prior Probabilities of Hypotheses

Calculate the probabilities $P[Y(t+1) = \mathrm{U}]$ and $P[Y(t+1) = \mathrm{D}]$. These probabilities will serve as the prior probabilities for the respective hypotheses. The outcome can be presented as follows:

- The prior probability of digit U is 0.353241
- The prior probability of digit D is 0.299462

4.2 Logistic Detector

In this segment, we will construct the detector using the logistic PDFs $L'_{\mathrm{U}}(x)$ and $L'_{\mathrm{D}}(x)$. We establish that $x \in (x_1, x_2)$ favors the null hypothesis, while $x < x_1$ or $x > x_2$ favors the alternative hypothesis. To construct the detector, it is necessary to solve the subsequent equation:

$$P[Y(t+1) = \mathrm{U} \mid X(t) = x] = P[Y(t+1) = \mathrm{D} \mid X(t) = x] \tag{5}$$

For the logistic PDFs, the equation can be expressed as follows:

$$L_U^{'}(x)P[Y(t+1) = U] = L_D^{'}(x)P[Y(t+1) = D] \tag{6}$$

By solving above equation, we can get the value of x_1 and x_2 as follows:

- The value of $x1$ for Logistic detector is -0.0286
- The value of $x2$ for Logistic detector is 0.0605

Following this, we plot $x = x_1$ and $x = x_2$ alongside the graph of $L'_{\mathrm{U}}(x)$ and $L'_{\mathrm{D}}(x)$ in Fig. 10.

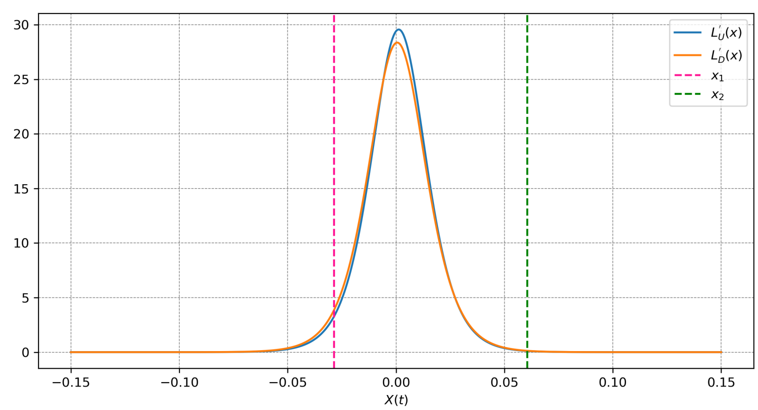

Fig. 10: The Detector with the Logistic PDFs

4.3 Gaussian Detector

We reiterate the previous step utilizing the Gaussian PDFs $g_{\mathrm{U}}(x)$ and $g_{\mathrm{D}}(x)$. To obtain the boundary of the Gaussian detector, it is necessary to solve the subsequent equation:

$$g_U(x)P[Y(t+1) = U] = g_D(x)P[Y(t+1) = D] \tag{7}$$

By solving the above equation, we can get the value of x_1 and x_2 as follows:

- The value of $x1$ for Gaussian detector is -0.0401
- The value of $x2$ for Gaussian detector is 0.0726

Subsequently, we plot $x = x_1$ and $x = x_2$ alongside the graph of $g_{\mathrm{U}}(x)$ and $g_{\mathrm{D}}(x)$ in Fig. 11.

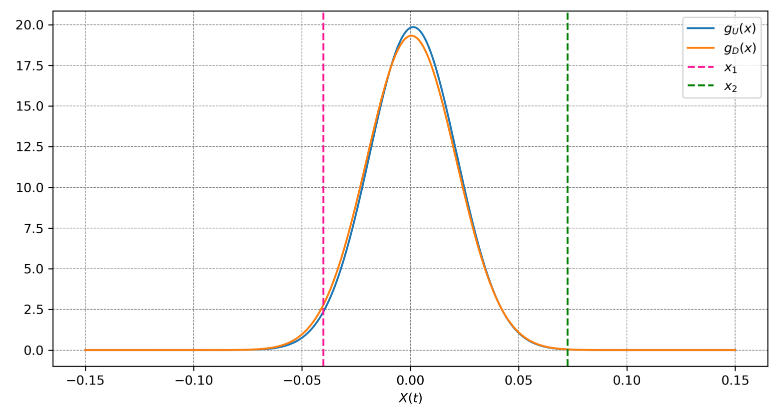

Fig. 11: The Detector with the Gaussian PDFs

5 Association Rule

While it is possible to consistently adhere to the Bayes detector and trade based on its recommendations, this approach would result in an excessive frequency of trading, leading to substantial losses due to transaction costs. As a result, we implement additional association rules to restrict the frequency of trading: we will only engage in trading when both the detector and the rules advocate for it. The general form of a k-day rule can be expressed as follows:

$$R_y^k : \{Y(t-k+1), Y(t-k+2), \ldots, Y(t)\} \to Y(t+1) = y \tag{8}$$

In this context, we opt for the Rule Power Factor (RPF) as the initial rule assessment method, which is defined as follows:

$$r_p = s_p c_p^2 \tag{9}$$

Here, s_p represents the past support of the rule, while c_p signifies the past confidence of the rule. The past support is defined as the proportion of instances where the rule was satisfied in the past. Similarly, past confidence is defined as the percentage of occurrences in which the rule was satisfied and the prediction was accurate in the past. To prevent the repetition of rules for distinct predictions, we opt for past confidence c_p as the secondary rule assessment method.

5.1 1-day Rule

Extract the optimal 1-day "upward rule" R_{U}^1 and "downward rule" R_{D}^1. The outcome can be presented as follows:

- The best rule for 1-day up is U with RPF: 0.3644
- The best rule for 1-day up is U with confidence: 0.3699
- The best rule for 1-day down is U with RPF: 0.3204
- The best rule for 1-day down is D with confidence: 0.3198

As the rule for 1-day downward movement is identical to the rule for 1-day upward movement when using RPF as the assessment method, we opt for confidence as the rule assessment approach. Consequently, we identify the best 1-day upward rule R_{U}^1 as U and the best 1-day downward rule R_{D}^1 as D.

5.2 5-day Rule

Subsequently, we reiterate the previous step to determine the two optimal 5-day rules R_{U}^5 and R_{D}^5. The outcome can be presented as follows:

- The best rule for 5-day up is HHDHH with RPF: 0.1277
- The best rule for 5-day up is HUDDD with confidence: 0.8571
- The best rule for 5-day down is UDUHH with RPF: 0.1241
- The best rule for 5-day down is UHDUD with confidence: 1.0000

Hence, by employing the RPF as the assessment method, we identify the best 5-day upward rule R_{U}^5 as HHDHH and the best 5-day downward rule R_{D}^5 as UDUHH.

6 Portfolio Management

Now that the preparation is complete, we can proceed to invest in the stock to evaluate the effectiveness of the Bayes detectors and association rules.

Let $M(t)$ represent the amount of money and $N(t)$ be the number of shares at the end of day t. As the portfolio consists solely of money and stock, its value is defined by the following equation:

$$V(t) = M(t) + N(t) \times S(t) \tag{10}$$

We initially set the portfolio to contain $M(0) = 100,000$ units (in the currency relevant to the data) and $N(0) = 0$ shares. We then commence trading at $t = 1$ in accordance with the following rules:

- If the Bayes detector predicts $Y(t+1) = \mathrm{U}$ and the antecedent of the best k-day upward rule R_{U}^k is observed on day t, purchase shares according to the following update rules:

$$\begin{cases} M(t) \leftarrow M(t) - m \\ N(t) \leftarrow N(t) + m/S(t) \end{cases} \quad \text{for } m = \gamma M(t) \tag{11}$$

– If the Bayes detector predicts $Y(t+1) = \mathrm{D}$ and the antecedent of the best k-day downward rule R_{D}^{k} is observed on day t, sell shares according to the following update rules:

$$\begin{cases} M(t) \leftarrow M(t) + nS(t) \\ N(t) \leftarrow N(t) - n \end{cases} \quad \text{for } n = \gamma N(t) \tag{12}$$

The parameter $\gamma \in (0,1)$ quantifies the trader's "greed." A greedier trader aims for higher earnings and, therefore, trades more per transaction.

For simplicity, we assume that $M(t)$ and $N(t)$ are real numbers with infinite precision, implying that there are no smallest units per transaction. This market model also unrealistically assumes that a stock's closing price for a day represents its price throughout the entire day and limits the trader to a maximum of one trade per day.

6.1 1-day Rule

We can now plot the portfolio performances for the two Bayes detectors obtained in Section 4, using $k = 1$ and $\gamma_0 = 0.1$, as shown in Fig. 12. Upon examining Fig. 12, it is evident that the Gaussian detector outperforms the Logistic detector when applying the 1-day rule.

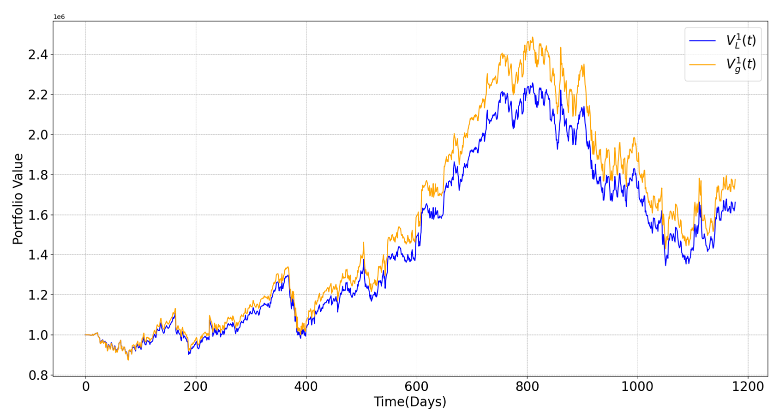

Fig. 12: Performances of portfolios with 1-day Rule

6.2 5-day Rule

Now, we can plot the portfolio performances for the two Bayes detectors obtained in Section 4, using $k = 5$ and $\gamma_0 = 0.1$, as depicted in Fig. 13.

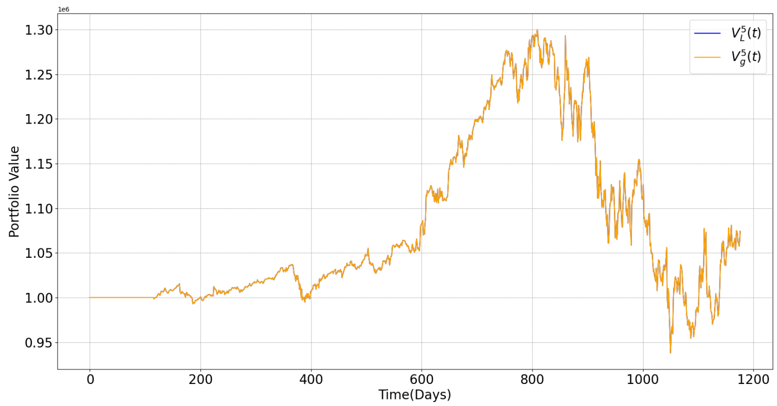

Fig. 13: Performances of portfolios with 5-day Rule

Upon examining Fig. 13, it becomes apparent that the portfolio performance of the Gaussian detector is equal to that of the Logistic detector when applying the 5-day rule. In fact, when using the 5-day rule, both detectors executed 7 identical transactions. The transaction history can be summarized as follows:

- Transaction on day 116: Buy 1795.65 shares at price 55.69
- Transaction on day 574: Buy 1010.23 shares at price 89.09
- Transaction on day 577: Buy 918.89 shares at price 88.15
- Transaction on day 746: Buy 532.45 shares at price 136.91
- Transaction on day 758: Buy 450.87 shares at price 145.52
- Transaction on day 841: Buy 408.88 shares at price 144.42
- Transaction on day 844: Buy 387.90 shares at price 137.00

6.3 Comparison

Investment portfolios are assessed utilizing four key metrics: mean, variance, minimum, and maximum values. The obtained results are presented as follows:

Table 1: Performance of Different Rules and Detectors

Rule	Detector	Mean	Variance	Min	Max
1-day	Logistic	1,445,213	163,905,453,432	880,440	2,256,585
1-day	Gaussian	1,534,026	218,417,697,685	873,949	2,485,364
5-day	Logistic	1,076,672	8,309,147,774	937,899	1,300,042
5-day	Gaussian	1,076,672	8,309,147,774	937,899	1,300,042

Upon examining the results, it becomes apparent that the portfolio utilizing the Gaussian detector outperforms the portfolio with the Logistic detector when the 1-day rule is applied. Consequently, the Gaussian detector will be employed exclusively in subsequent sections of this study.

Additionally, it is worth noting that the portfolios implementing the 1-day rule demonstrate significantly better performance compared to those following the 5-day rule. This observation may be attributed to the increased volatility of stock prices over shorter timeframes, where more frequent transactions have the potential to yield higher profits.

7 Transaction Cost

Markets typically impose certain execution fees on transactions. Consequently, we adjust the buying scheme from Eq. 11 to Eq. 13 and the selling scheme from Eq. 12 to Eq. 14 by incorporating a tax parameter, denoted as ξ. This modification aims to account for the plausible suppression of trading frequency due to transaction costs.

$$\begin{cases} M(t) \leftarrow M(t) - m \\ N(t) \leftarrow N(t) + (1-\xi)m/S(t) \end{cases} \quad \text{for } m = \gamma M(t) \tag{13}$$

$$\begin{cases} M(t) \leftarrow M(t) + (1-\xi)nS(t) \\ N(t) \leftarrow N(t) - n \end{cases} \quad \text{for } n = \gamma N(t) \tag{14}$$

7.1 Comparison with different k-day rules

The resulting portfolio values, denoted as $V^1(t;\xi)$ with $k=1$ and $V^5(t;\xi)$ with $k=5$, are depicted in Fig. 14.

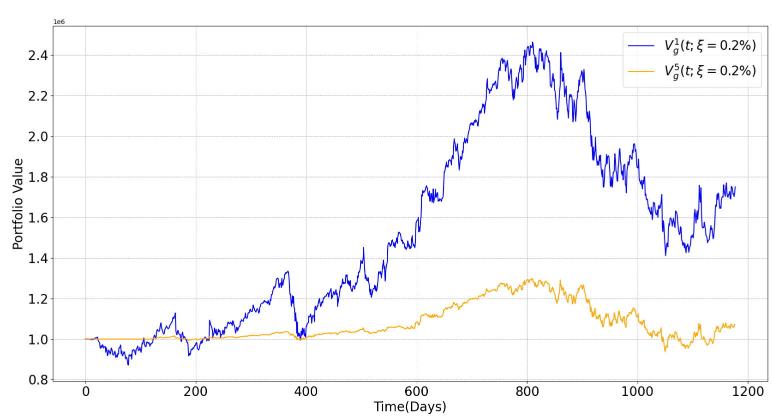

Fig. 14: Performances of portfolios with different k-day rules

7.2 Comparison with different tax rates

The resultant values of $V^1(t;\xi)$ and $V^5(t;\xi)$ are plotted in Fig. 15 for three distinct tax rates: $\xi = 0.1\%$, 0.2%, and 0.5%

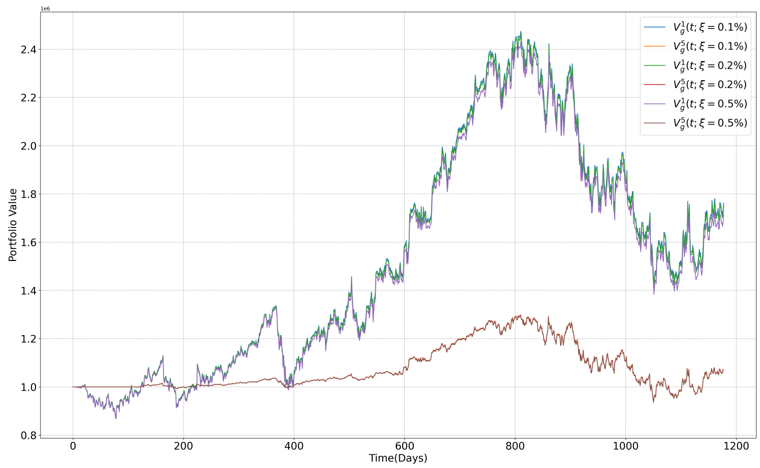

Fig. 15: Performances of portfolios with different tax rates

The number of transactions in each case is as follows:

Table 2: Number of Transactions for Different Cases

k	ξ	**Number of Transactions**
1	0.1%	468
5	0.1%	7
1	0.2%	468
5	0.2%	7
1	0.5%	468
5	0.5%	7

Upon examining Fig. 15, it becomes evident that the portfolio implementing the 1-day rule outperforms the portfolio adhering to the 5-day rule. Consequently, the 1-day rule will be employed exclusively in the subsequent sections of this study.

8 Risk-Free Interest

We further assume that $M(t)$ grows without risk, as the trader has deposited their funds in a bank. Consequently, at the start of each day, we first update the following:

$$M(t) \leftarrow M(t) \times (1 + r) \tag{15}$$

Here, r represents the daily interest rate. In this section, we initially set the parameters as $r = 0.001\%$, $\xi = 0.2\%$, and $\gamma = \gamma_0$.

8.1 Trade with Risk-Free Interest

The resultant portfolio's value $V(t; r)$ is shown in Fig 16.

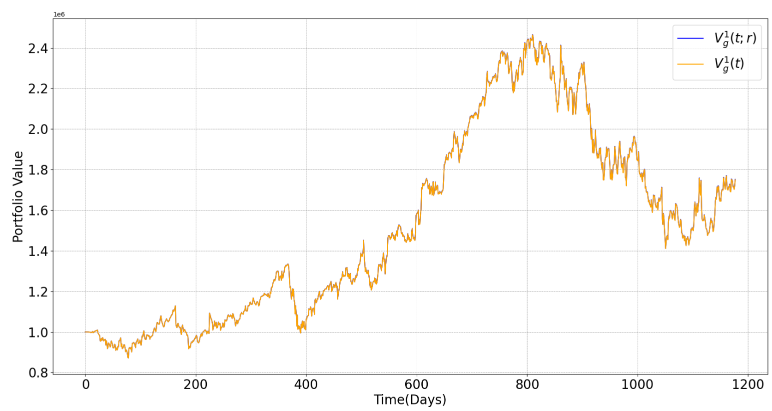

Fig. 16: Performances of portfolios with risk-free interest

8.2 Ratio Compared to Benchmark

If the trader does not trade at all, the portfolio's value reduces to

$$V_b(t; r) = M(0) \times (1 + r)^t \tag{16}$$

which can serve as the portfolio's benchmark value. We plot the ratio $\rho(t; r) \equiv V(t; r)/V_b(t; r)$ in Fig. 17.

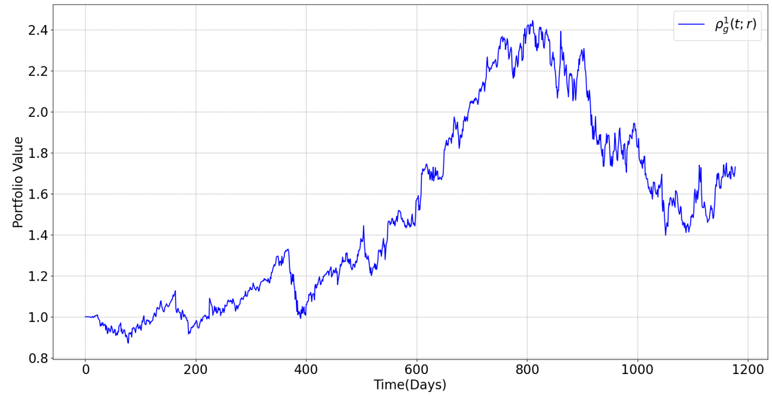

Fig. 17: Ratio of portfolio's value to benchmark value

8.3 Comparison with Different Interest Rates

Repeat the experiment with $r = 0.005\%$ and 0.01%, whereas Section 7 essentially uses $r = 0$ and plot the ratio $\rho(t;r)$ in Fig. 18. Upon examining Fig. 18, it

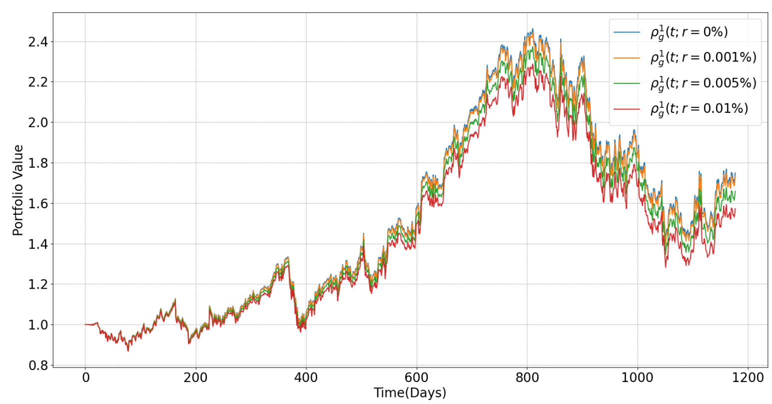

Fig. 18: Performances of portfolios with different free-interest rates

becomes evident that the portfolio with $r = 0\%$ outperforms those with $r = 0.001\%$, $r = 0.005\%$, and $r = 0.01\%$. This observation concludes that $\rho(t;r)$ exhibits a decreasing trend as the value of r increases.

9 Greed

Finally, investigate the effect of γ. Consider $\xi = 0.2\%$ and $r = 0.001\%$.

9.1 Comparison with Different Greeds

In Fig. 19, we present a visual representation of the portfolio value $V(t;\gamma)$, resulting from trades executed for twenty distinct values of γ, including $\gamma \in 0.1, 0.3, 0.5, 0.7, 0.9$. 19.

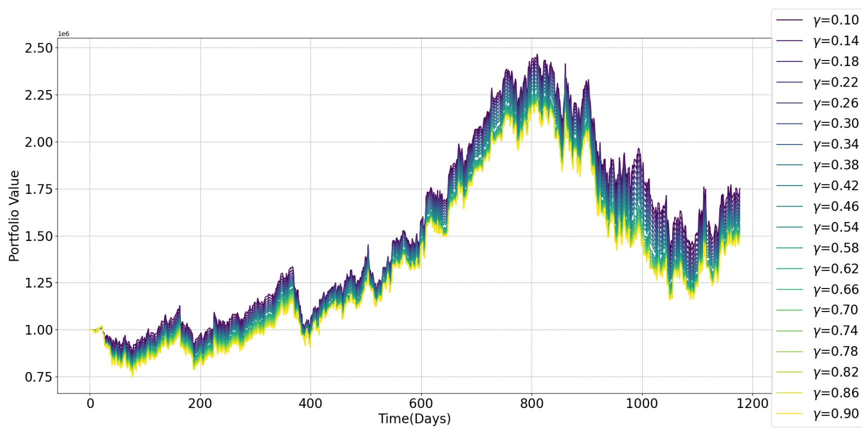

Fig. 19: Performances of portfolios with Greeds

9.2 The Effect of Greed

Fig. 20 presents a comprehensive visualization of the portfolio's final values $V(\max t;\gamma)$, their peak values $\max_t V(t;\gamma)$, and their time-averaged values $\langle V(t;\gamma)\rangle$ in relation to the parameter γ. A thorough examination of Fig. 20 reveals that $V(\max t;\gamma)$, $\max_t V(t;\gamma)$, and $\langle V(t;\gamma)\rangle$ all demonstrate a diminishing trend as γ increases. Based on these observations, we can infer that an excessive emphasis on immediate gains, as characterized by a higher γ value, may adversely impact the overall performance of the trading strategy.

10 Efficient Frontier

Although employing $m = \gamma M(t)$ and $n = \gamma N(t)$ for each transaction may appear somewhat arbitrary, this approach can be justified by considering the efficient frontier, thereby generalizing the investment scheme.

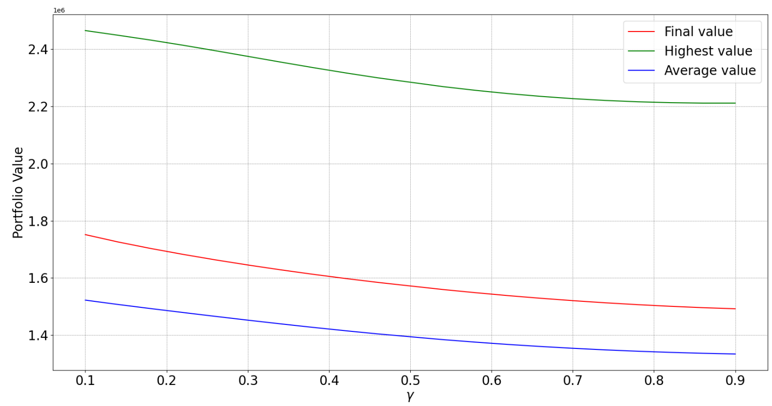

Fig. 20: $V(\max t; \gamma)$, $\max_t V(t;\gamma)$ and $\langle V(t;\gamma)\rangle$ versus γ

Let u, u_1, and u_2 represent the expected returns of the portfolio, $M(t)$, and $S(t)$. Since $M(t)$ is risk-free, $u_1 \le u_2$ should hold in a stable market; otherwise, risky investments would not be rewarded. Consequently, both the risk and return of a portfolio are minimized when it consists solely of money, and maximized when it consists exclusively of the stock.

Immediately prior to a transaction on day t, the portfolio's return amounts to $u = Au_1 + (1-A)u_2$, where $A = M(t)/V(t)$ is the proportion of money. If a buy signal is observed, an investor spends $\widetilde{m} = \gamma_{\mathrm{U}} M(t)$ to purchase $(1-\xi)\widetilde{m}/S(t)$ shares, altering u to $u_{\mathrm{U}} = A_{\mathrm{U}} u_1 + (1-A_{\mathrm{U}})u_2$. Conversely, if a sell signal is observed, an investor sells $\tilde{n} = \gamma_{\mathrm{D}} N(t)$ shares to earn $(1-\xi)\tilde{n}S(t)$, modifying u to $u_{\mathrm{D}} = A_{\mathrm{D}} u_1 + (1-A_{\mathrm{D}})u_2$.

In this context, the investor adjusts their greed level to either γ_{U} or γ_{D} for each transaction, with both levels depending on the investor's intrinsic greed, $\gamma \in (0,1)$. A greedier investor buys more when making a purchase, thus pushing their portfolio further towards a stock-only portfolio along the efficient frontier. This relationship can be expressed as $\begin{cases} \gamma \to 1 \Rightarrow u_{\mathrm{U}} \to u_2 \\ \gamma \to 0 \Rightarrow u_{\mathrm{U}} \to u \end{cases}$, and can be modeled with the following equation:

$$u_{\mathrm{U}} = \gamma(u_2 - u) + u \tag{17}$$

Similarly, a greedier investor sells more when selling, thereby pushing their portfolio further towards a money-only portfolio. This behavior can be described by:

$$u_{\mathrm{D}} = \gamma(u_1 - u) + u \tag{18}$$

10.1 Formula Derivation of u_U and u_D

First, we aim to express A_U in terms of $\gamma_U, V(t), M(t), \xi$. Based on the previous discussion, we have:

$$\begin{aligned}
&\tilde{m} = \gamma_U M(t) \\
&V(t) = M(t) + N(t)S(t) \\
&M(t+1) = (1-\gamma_U)M(t) \\
&N(t+1) = N(t) + (1-\xi)\tilde{m}/S(t)
\end{aligned}$$

From these equations, we can derive the formula for $U(t+1)$:

$$\begin{aligned}
V(t+1) &= M(t+1) + N(t+1)S(t) \\
&= (1-\gamma_U)M(t) + N(t)S(t) + (1-\xi)\gamma_U M(t) \\
&= V(t) - \xi\gamma_U M(t)
\end{aligned}$$

Then, we obtain the formula for A_U:

$$A_U = M(t+1)/V(t+1) = \frac{(1-\gamma_U)M(t)}{V(t) - \xi\gamma_U M(t)} \tag{19}$$

Similarly, for A_D, we have:

$$\begin{aligned}
&\tilde{n} = \gamma_D N(t) \\
&V(t) = M(t) + N(t)S(t) \\
&N(t+1) = (1-\gamma_D)N(t) \\
&M(t+1) = M(t) + (1-\xi)\gamma_D N(t)S(t) \\
V(t+1) &= M(t+1) + N(t+1)S(t) \\
&= V(t) - \xi\gamma_D N(t)S(t)
\end{aligned}$$

Then, we can derive the formula for $U(t+1)$:

$$\begin{aligned}
V(t+1) &= M(t+1) + N(t+1)S(t) \\
&= V(t) - \xi\gamma_D N(t)S(t)
\end{aligned}$$

Next, we derive the formula for $1 - A_D$:

$$1 - A_D = N(t+1)S(t)/V(t+1) = \frac{(1-\gamma_D)N(t)S(t)}{V(t) - \xi\gamma_D N(t)S(t)} \tag{20}$$

Second, we express A_U/A in terms of γ by solving the following equations:

$$\begin{cases} A_U u_1 + (1-A_u)\, u_2 = \gamma\,(u_2 - u) + u \\ u = Au_1 + (1-A)u_2 \end{cases}$$

From these equations, we derive the formula:

$$\frac{A_u}{A} = 1 - \gamma \tag{21}$$

Similarly, by solving the following equations:

$$\begin{cases} A_D u_1 + (1 - A_D)\, u_2 = \gamma\,(u_1 - u) + u \\ u = A u_1 + (1 - A) u_2 \end{cases}$$

We obtain the formula:

$$\frac{1 - A_D}{1 - A} = 1 - \gamma \tag{22}$$

Finally, we derive the formula for γ_U by solving Eq. 19 and Eq. 21, and the formula for γ_D by solving equations Eq. 20 and Eq. 22:

$$\gamma_U = \frac{\gamma V(t)}{V(t) - \xi M(t)(1-\gamma)} \tag{23}$$

$$\gamma_D = \frac{\gamma V(t)}{V(t) - \xi N(t) S(t)(1-\gamma)} \tag{24}$$

10.2 Comparison of Efficient Frontier to Usual Investment Scheme

It turns out that $\gamma = \gamma_{\mathrm{U}} = \gamma_{\mathrm{D}}$ for a small tax ξ. This justifies trading with $m = \gamma M(t)$ and $n = \gamma N(t)$. Now consider an absurdly heavy tax $\xi = 50\%$ with $r = 0.001\%$ and $\gamma = \gamma_0$. We plot the resultant portfolio's value $V(t)$ when trade with m and n as usual and the resultant portfolio's value $\tilde{V}(t)$ when trading with $\widetilde{m}$ and $\tilde{n}$ in Fig. 21. By observing Fig. 21, we can see that the usual scheme

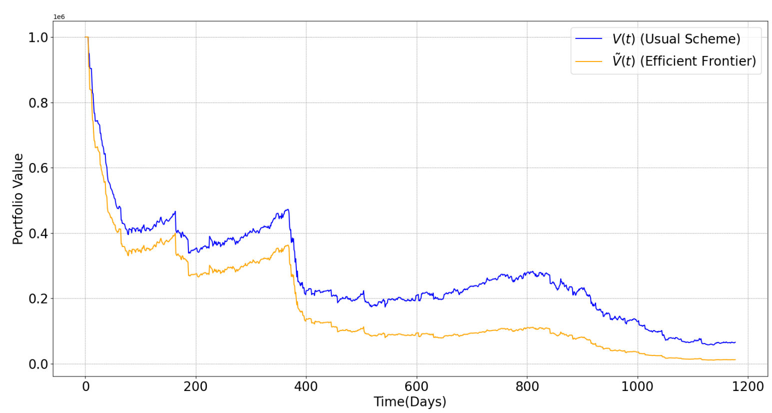

Fig. 21: Comparison of Efficient Frontier to Usual Investment Scheme

performs better than the efficient frontier scheme with the given parameters.

11 Adaptive Greed

Thus far, our analysis has employed a constant greed factor throughout the trading process. We now propose to examine an adaptive greed strategy that dynamically responds to market conditions. The forthcoming investigation will utilize the framework delineated in Sections 6 to 8, deviating from the previously derived approach.

11.1 Posterior analysis

We have obtained twenty portfolios $\{V(t;\gamma)\}$ for twenty choices of γ in Section 9 using $\xi = 0.2\%$ and $r = 0.001\%$. Then we plot

$$\gamma_i^* = \underset{\gamma}{\text{argmax}} \left[\frac{V\left(t_i;\gamma\right)}{V\left(t_i-1;\gamma\right)} \right]$$

against i in Fig. 22, where the i th transaction happens at $t = t_i$.

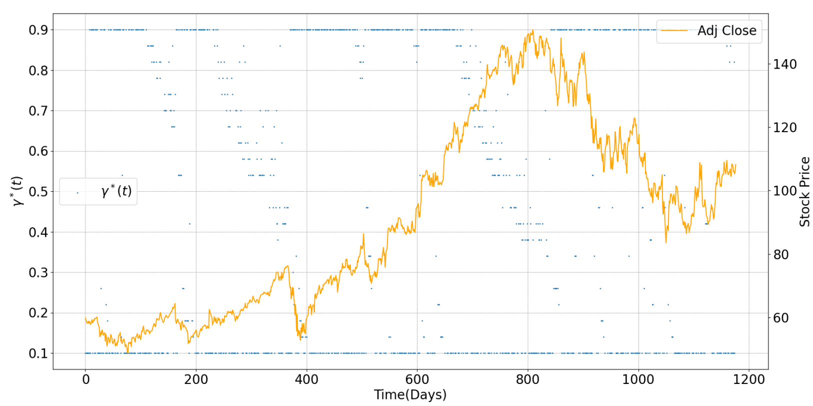

Fig. 22: Plot of γ_i^* against i

Upon analyzing Fig. 22, it becomes apparent that the optimal greed factor, γ_i, exhibits a positive correlation with the rising stock price and a negative correlation with the falling stock price. This observation aligns with the established investment principle of increasing share purchases when the stock price is on an upward trajectory and intensifying sales when the stock price is experiencing a decline. Subsequently, we proceed to execute trades with the adaptive greed factor, $\gamma = \gamma_i$, during the i-th transaction. A comparison of the resultant portfolio values, $V^*(t)$ and $V(t)$, is presented in Fig. 23.

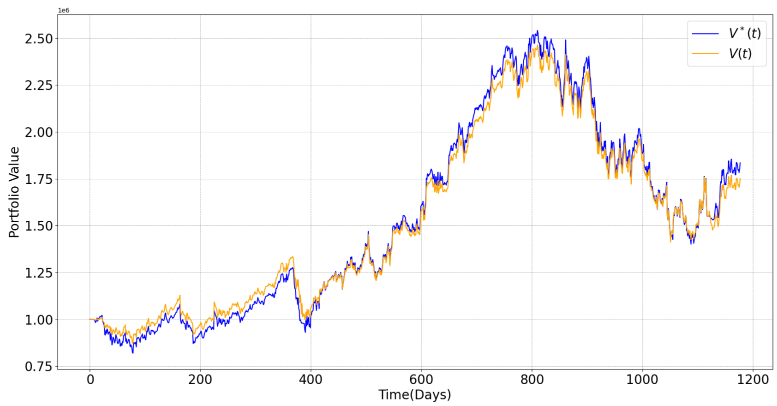

Fig. 23: Comparison of $V^*(t)$ and $V(t)$

The plot in Fig. 23 facilitates an evaluation of the performance of the adaptive greed strategy relative to the constant greed strategy. By scrutinizing the results, we can ascertain the efficacy of the adaptive greed approach in optimizing the portfolio's value under a diverse range of market conditions.

11.2 Prior analysis

Practically, γ_i^* is useless because it is obtained a posteriori, while we need to decide γ before trading. Since it is hard to choose its value from a real interval $(0, 1)$, we simplify the problem to choosing between two values $\{\gamma_A, \gamma_C\}$: before each investment, we decide whether it is better to use an aggressive greed γ_A or a conservative greed γ_C.

Consider a scenario where Alice, Bob, and Charlie each have their respective bankers. Alice's investment strategy is characterized by a greed factor of $\gamma_A = 0.7$, while Charlie's strategy is marked by a greed factor of $\gamma_C = 0.3$. Bob aims to establish an equilibrium between these divergent approaches by determining the appropriate instances to invest with $\gamma_B = \gamma_A$ and when to allocate funds with $\gamma_B = \gamma_C$.

In order to select an adaptive greed factor through technical analysis, one possible method involves employing a moving average crossover strategy. This technique entails the utilization of two distinct moving averages, characterized by a shorter and a longer time period, respectively. A bullish signal is generated when the shorter moving average surpasses the longer moving average, prompting an increase in the greed factor to $\gamma_A = 0.7$. Conversely, a bearish signal arises when the shorter moving average falls below the longer moving average, necessitating a reduction in the greed factor to $\gamma_C = 0.3$.

In the present analysis, the shorter time period is designated as a 10-day interval, while the longer time period encompasses a span of 60 days. When the portfolio series' length falls short of 60 days, a conservative greed factor of $\gamma_C = 0.3$ is adopted to accommodate the limited data availability.

Execute trades for Alice, Bob, and Charlie with transaction costs $\xi = 0.2\%$ and interest rate $r = 0.001\%$. Depict the respective portfolio values $V_A(t), V_B(t)$, and $V_C(t)$ in Fig. 24.

Upon examination of Fig. 24, it becomes evident that Bob's portfolio value outperforms those of Alice and Charlie, indicating the effectiveness of the moving average crossover strategy in optimizing investment outcomes.

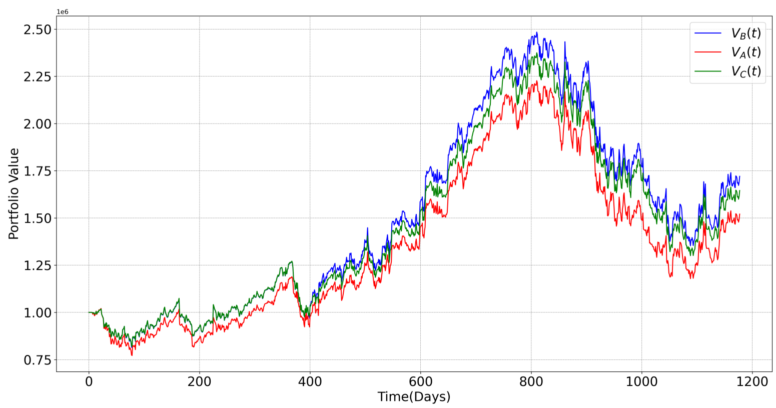

Fig. 24: Performance of Alice, Bob, and Charlie's portfolios

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